

## Path analysis model estimates using generalized method of moment (Case study: Maternal mortality in the Province of East Java)

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### Abstract

Health sector development directed to reach international commitment who poured in millennium development goals (MDGs). Reducing child mortality and to improve maternal health is the goal of the mdgs associated directly with health namely to lower maternal mortality rate (MMR). MMR high in a region basically describes degrees health low public and potentially cause a setback economic and social in the level of household appliances the community and nasional. One of effort to overcome these problems were do an analysis about factor-factor affecting maternal mortality. One method used in this research is by using path analysis. The estimation method used to estimate is the method of moments and maximum likelihood estimation. This study used estimates of Generalized Method of Moment (GMM) as an extension of the method of moments. GMM estimation is used to exploit information form of the condition of the population moment. The results obtained, parameter estimation path analysis done gradually because consists of two structural models. The application in the case of maternal mortality in East Java suggests that the variable percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry the K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted by dukun to contribute to maternal mortality through a variable percentage pregnant women at high risk/complication.

**Keywords:** GMM estimation, path analysis, maternal mortality

### 1. Introduction

Health development geared to achieving international commitments, as outlined in the MDGs. Reduce child mortality and improve maternal health is the Millennium Development Goals directly related to health, namely to reduce MMR and Infant Mortality Rate (IMR), which is an indicator of the quality of public health services in a country. Maternal mortality is the death of every woman during pregnancy, childbirth or within 42 days after the end of the pregnancy from any cause, regardless of the age and location of the pregnancy, by any cause related to or aggravated by pregnancy or its handling but not by accident or incidental (factor accidental). AKI is high in a region basically describes a low degree of public health and potentially causing economic and social deterioration in the level of the household, community and national levels. However, the biggest impact of maternal mortality in the form of decreased quality of life of infants and children caused shock in the

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family and further affect the development of the child. Statistical analysis is used to solve many problems in the study who used regression analysis and regression with spatial effects. The aim was to determine the relationship between variables and causal relationship of the variables in the study. Therefore, researchers are interested in using different methods in solving the problems of maternal mortality. The method used in this research is by using path analysis.

Path analysis is part of a regression analysis that can be used to analyze the causal relationship between one variable with another variable. In contrast to the regression analysis where the influence of free variable and dependent form of direct influence, in the analysis of exogenous variables influence the path of the endogenous variable can be either direct or indirect influence. In regression analysis and path analysis will produce a model equation that will require estimation of the model parameters obtained. One method of estimation generalization of the Moment method is a method of GMM.

GMM was first introduced by Lars Peter Hansen in 1982. GMM is obtained by minimizing the weighted sum of the squares of the condition of the sample moments. The weighting matrix is an important element in the method of GMM, which is obtained through the variance-covariance matrix of the condition of the sample moments (Wooldridge, 2001). Based on the research that has been described, rarely found parameter estimation using GMM in the social field, including the path analysis model. Therefore, the researchers intend to conduct research on the estimation model path analysis by using GMM on maternal mortality in East Java.

## 2. Materials and methods

### 2.1 Path analysis

Path analysis is a development technique of multiple linear regression. This technique is used to examine the contributions shown by the path coefficient for each path diagram of causal relationships between variables  $X_1, X_2, X_3$  to  $Y$ . Analysis of pathways including models similar to the shape of the regression analysis, factor analysis, canonical correlation analysis, discriminant analysis, as models in the multivariate analysis of variance and covariance analysis (MANOVA, ANOVA, ANCOVA) (Wright, 1921). Figure 1 presents an example of the path diagram. For Figure 1, structural equation can be written as follows

$$Y = \gamma_{11}X_1 + \gamma_{12}X_2 + \varepsilon.$$

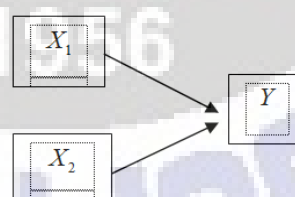


Figure 2. Example path diagram.

To evaluate the suitability of the model it is used a measure or criteria of goodness of fit. For the first action taken is to evaluate whether the data used to meet the assumptions. If these assumptions are met, then the model can be tested via various test methods that will be described in this section. First will be described here regarding the evaluation of assumptions that must be met. The assumptions that must be met in data collection and

processing procedures were analyzed by modeling is normality and linearity, outlier and Multicollinearity.

## 2.2 Parameter estimation

Population parameter is usually the price is not known, so it needs to be estimated based on the observation of sample data. The accuracy of the estimated parameters depending on the sample size and methods used for parameter estimation. The statistics are calculated from the sample used to estimate a population parameter called the estimator. A good estimator has properties: no bias, consistent, efficient and sufisien. Suppose the statistics used to estimate population parameters,  $\theta$ , is called a point estimator for  $\theta$ , denoted. Parameter estimation, there are two kinds namely, the point estimate and interval estimation.

### Method of moment

Method of moments (MM) who created Karl Pearson in 1800 is the oldest method in the determination of the point estimator. The main idea of the method of moments is evened specific sample characteristics such as mean and variance for the expected values of the corresponding population and then completing the resulting equation to obtain the approximate value of the unknown parameter. Suppose  $X_1, X_2, \dots, X_n$  a random sample of the population with a probability function  $f(x|\theta)$ . The  $k$ -th moment of the sample is given by

$$M_k = \frac{1}{n} \sum_{i=1}^n X_i^k.$$

Suppose that a population with a probability function  $f(x|\theta)$  to the population of the moment- $k$  can be defined as:

$$\mu_k(\theta_1, \theta_2, \dots, \theta_n) = E(X^k).$$

So the moment estimator  $\hat{\theta}_{MM}$  for the parameters  $\theta$  obtained from completing the simultaneous equations as follows.

$$\begin{aligned} \mu_1 = M_1 \text{ or } E(X) &= \frac{1}{n} \sum_{i=1}^n X_i \\ \mu_2 = M_2 \text{ or } E(X^2) &= \frac{1}{n} \sum_{i=1}^n X_i^2 \\ &\vdots \\ \mu_k = M_k \text{ or } E(X^k) &= \frac{1}{n} \sum_{i=1}^n X_i^k \end{aligned}$$

Examples:

1) Suppose that  $X_1, X_2, \dots, X_n$  a random sample taken from the population exponentially with probability functions as follows:

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} I_{(0, \infty)}(x),$$

then get a moment's estimator is to calculate the expected value of

$$E(X) = \int_0^{\infty} x \left( \frac{1}{\theta} e^{-\frac{x}{\theta}} \right) dx = \theta.$$

Equation

$$\mu_1(\hat{\theta}) = M_1$$

gives

$$\hat{\theta}_{MM} = \frac{1}{n} \sum_{i=1}^n X_i = \bar{X}.$$

### GMM

GMM is a method used to obtain the parameter estimation of statistical models. Generalized Method of Moments is a form of generalization of Method of Moment developed by Lars Peter Hansen in 1982. The estimation method is GMM take the concept of the method of moments, moment method whereby if at the moment the conditions amount equal to the number of parameters to be estimated, whereas for GMM method the number of moment conditions is greater than or equal to the number of parameters to be estimated. The application of the concept of GMM, the necessary condition of the moment of the function  $f(Y, \theta)$  with the following notations:

$$m(\theta_0) = E[f(w_t, \theta_0)] = 0,$$

where  $\theta$  as a vector of parameters with size  $K \times 1$ ,  $f(\cdot)$  as the vector dimension of the functions  $M$ ,  $w_t$  as a model of variables, and  $E[f(w_t, \theta_0)]$  represents the condition of the order moments of  $K$  and  $M$ . Suppose  $\{w_t\}_{t=1}^N$  an i.i.d sample from a distribution having scale parameter  $\theta_{K \times 1} = [\theta_1, \theta_2, \dots, \theta_K]'$  where the function  $f(w_t, \theta)$  dimension  $m \times 1$ :

$$f(w_t, \theta) = \begin{bmatrix} f_1(w_t, \theta) \\ f_2(w_t, \theta) \\ \vdots \\ f_M(w_t, \theta) \end{bmatrix}.$$

The conditions of the intended moment  $M$  is

$$E[f(w_t, \theta)] = E \begin{bmatrix} f_1(w_t, \theta) \\ f_2(w_t, \theta) \\ \vdots \\ f_M(w_t, \theta) \end{bmatrix} = \begin{bmatrix} E[f_1(w_t, \theta)] \\ E[f_2(w_t, \theta)] \\ \vdots \\ E[f_M(w_t, \theta)] \end{bmatrix} = 0.$$

The next set of samples analogy of the conditions of the moment as follows.

$$m_n(\theta) = \frac{1}{N} \sum_{t=1}^N f(w_t, \theta) = \begin{bmatrix} \frac{1}{N} \sum_{t=1}^N f_1(w_t, \theta) \\ \frac{1}{N} \sum_{t=1}^N f_2(w_t, \theta) \\ \vdots \\ \frac{1}{N} \sum_{t=1}^N f_M(w_t, \theta) \end{bmatrix} = 0.$$

After determining the conditions of the moment and the analogy of the samples, the procedures of the next GMM method is the provision of a weighted matrix to solve systems of equations. In determining the weighting matrix, the weighting matrix is assumed to be positive definite and constant-value (Hansen, 1982) size  $m \times m$  as follows.

$$\mathbf{V}_n \xrightarrow{p} \mathbf{V}.$$

After the weighting matrix is determined, the procedure of GMM method is to define a function object  $Q_n(\theta)$  with the following notations:

$$Q_n(\theta) = m_n(\theta)' \mathbf{V}_n^{-1} m_n(\theta).$$

The next procedure is to use the GMM estimator as a solution to minimize the object function  $Q_n(\theta)$  with the following notations:

$$\hat{\theta}_{GMM} = \arg \min_{\theta} \{Q_n(\theta)\} = \arg \min_{\theta} \{m_n(\theta)' \mathbf{V}_n^{-1} m_n(\theta)\}.$$

In the estimation process by using the GMM method for the selection weighting matrix used has no special provision, but is worth the weighting matrix positive definite so that the differences in the election then generates different asymptotic distribution. The weighting matrix selection considerations necessary to minimize the variance of GMM estimators (Hansen, 1982).



Optimization GMM estimator obtained by using the optimal weighting matrix in which required two procedures, namely:

- 1) The initial procedure to be done is to estimate the GMM estimator with weighted matrix  $V_n = I_z$ , as a solution to minimize process:

$$\hat{\theta}_{GMM,1st\_step} = \arg \min_{\theta} \{m_n(\theta)' I_z^{-1} m_n(\theta)\} = \arg \min_{\theta} \{m_n(\theta)' m_n(\theta)\},$$

where  $\hat{\theta}_{GMM,1st\_step}$  the GMM estimator obtained through the weighting matrix identity  $I_z$ .

- 2) The next procedure is defined for the second  $V_n = \Omega$ , weighting matrix then used  $\hat{\theta}_{GMM,1st\_step}$  to calculate the estimated variance of  $f(w_t, \theta)$  with the following notations:

$$\Omega = \frac{1}{N} \sum_{t=1}^N [f(w_t, \hat{\theta}_{GMM,1st\_step}) f(w_t, \hat{\theta}_{GMM,1st\_step})']$$

so be solving the minimization of the following equation:

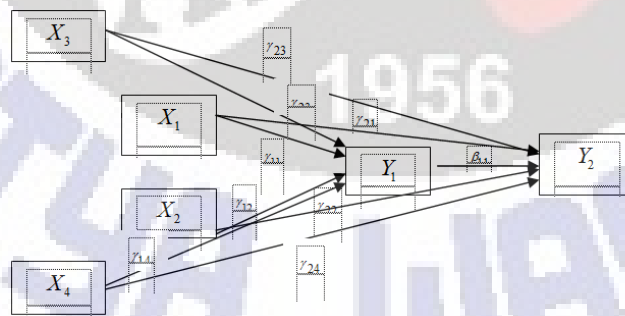
$$\hat{\theta}_{OptimalGMM} = \arg \min_{\theta} \{m_n(\theta)' \Omega^{-1} m_n(\theta)\}.$$

### 2.3 Method

The data used in this research is secondary data obtained in 2013 from the East Java Provincial Health Office and the data of the National Socioeconomic Survey (Susenas) East Java province in 2013. The observation unit used in this study consisted of 29 districts and 9 Cities in East Java province.

Variables used in this study consists of two variables: the endogenous variable in this case is the amount of Maternal Mortality While exogenous variable is a variable percentage of pregnant women at high risk/complications handled every district/city, the percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted by shaman each district/city.

Maternal deaths due to pregnancy, childbirth and post-partum is already a lot of peeled and discussed the causes and measures to overcome them. Yet it seems the various efforts that have been made by the government is still not able to accelerate the decline in MMR as expected. Classic conceptual framework that is still used in discussing determinants of maternal mortality is presented by McCarthy & Maine (1992).



$$\text{Equation 1: } Y_1 = \gamma_{11}X_1 + \gamma_{12}X_2 + \gamma_{13}X_3 + \varepsilon_1$$

$$\text{Equation 2: } Y_2 = \beta_{11}Y_1 + \gamma_{21}X_1 + \gamma_{22}X_2 + \gamma_{23}X_3 + \gamma_{24}X_4 + \varepsilon_2$$

From the second equation will be sought variables that influence directly or indirectly on the number of maternal deaths.

In general, the steps in this research are as follows:

- 1) Identification of research data.
- 2) Modeling path analysis.
- 3) Estimate the parameters by using the method GMM according to the following steps:
  - a) Determine the condition of the moment.
  - b) Determining analogy sampel.
  - c) Selection of weighting matrix.
  - d) Minimize the object function using GMM estimator.
- 4) Apply the estimate obtained no existing data.
- 5) Interpretation of coefficients and parameters of conclusion.

### 3. Results and discussion

Based on this research path diagram formed two structural equation, the equation is as follows:

$$\text{Equation 1: } Y_1 = \gamma_{11}X_1 + \gamma_{12}X_2 + \gamma_{13}X_3 + \varepsilon_1$$

$$\text{Equation 2: } Y_2 = \beta_{11}Y_1 + \gamma_{21}X_1 + \gamma_{22}X_2 + \gamma_{23}X_3 + \gamma_{24}X_4 + \varepsilon_2$$

#### Structural equation 1

If the observed total  $N$  observations, then the equation can be written as

$$y_1 = X_i'\gamma + u_1,$$

where  $i = 1, 2, \dots, N$  and

$y_1$  = the percentage of pregnant women at high risk/complications handled,

$u_1$  = error in pregnant women at high risk/complications handled,

$X_i'$  = vector of exogenous variables (variable percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry the K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted by shaman) with size  $\gamma$  = vector regression coefficients measuring.

In this case,  $\gamma$  is a unique solution to the equation moments of the population. Population moment equation is written as follows:

$$E(g(\gamma)) = E(u_1) = E(y_1 - X_i'\gamma) = 0$$

corresponding to the moment of the sample

$$\bar{g} = \frac{1}{N} \sum_{i=1}^N y_1 - X_i'\gamma.$$

Then built a function of GMM which is a quadratic function of the moment of the sample. The functions are as follows.

$$J(\hat{\gamma}) = \|\bar{g}(\hat{\gamma})\|^2 = \bar{g}(\hat{\gamma})\hat{W}\bar{g}(\hat{\gamma}) = \left[ \frac{1}{N} \sum_{i=1}^N (y_1 - X_i'\gamma) \right]' \hat{W} \left[ \frac{1}{N} \sum_{i=1}^N (y_1 - X_i'\gamma) \right],$$

where  $\hat{W}$  is the weight estimator. Sample moments quadratic function is translated as follows.

$$\begin{aligned} J(\hat{\gamma}) &= \left[ \frac{1}{N} \sum_{i=1}^N (y_1 - X_i'\gamma) \right]' \hat{W} \left[ \frac{1}{N} \sum_{i=1}^N (y_1 - X_i'\gamma) \right] \\ &= \left( \frac{1}{N} \sum_{i=1}^N y_1' \right) \hat{W} \left( \frac{1}{N} \sum_{i=1}^N y_1 \right) - 2 \left( \frac{1}{N} \sum_{i=1}^N X_i\gamma' \right) \hat{W} \left( \frac{1}{N} \sum_{i=1}^N y_1 \right) \\ &\quad + \left( \frac{1}{N} \sum_{i=1}^N X_i\gamma' \right) \hat{W} \left( \frac{1}{N} \sum_{i=1}^N X_i\gamma \right). \end{aligned}$$

GMM estimators to be obtained by minimizing the quadratic function  $J(\hat{\gamma})$ . Therefore,

$$\frac{\partial J(\hat{\gamma})}{\partial \hat{\gamma}} = 0$$

$$\hat{\gamma} = \frac{\left(\frac{1}{N}\sum_{i=1}^N X_i'\right)\widehat{W}\left(\frac{1}{N}\sum_{i=1}^N y_2\right)}{\left(\frac{1}{N}\sum_{i=1}^N X_i\right)\widehat{W}\left(\frac{1}{N}\sum_{i=1}^N X_i'\right)}$$

The estimation results are the result of the first estimate for the structural equation. Next we will estimate for the second structural equation model parameters.

### Structural Equation 2

By using the similar step with structural equations 1, it can be produced:

$$\hat{\gamma} = \frac{\left(\frac{1}{N}\sum_{i=1}^N y_2'\right)\widehat{W}\left(\frac{1}{N}\sum_{i=1}^N X_i^*\right) - \left(\frac{1}{N}\sum_{i=1}^N y_{11}'\widehat{\gamma}_1'\right)\widehat{W}\left(\frac{1}{N}\sum_{i=1}^N X_i^*\right)}{\left(\frac{1}{N}\sum_{i=1}^N X_i^*\right)\widehat{W}\left(\frac{1}{N}\sum_{i=1}^N X_i^*\right)}$$

This analysis will be discussed on the estimated parameters of the variable percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry the K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted by shaman each district/city influential indirectly through a variable percentage pregnant women are at high risk/complications handled every district/city against maternal mortality in East Java.

$$\text{Wanita hamil yang beresiko tinggi} = \begin{cases} -5.1285e - 01 \text{ Program K1} \\ 6.8737e - 01 \text{ Pemberian tablet FE1} \\ -2.2890e - 01 \text{ Rumah tangga ber-PHBS} \\ -5.2686e - 01 \text{ Persalinan bantuan dukun} \end{cases}$$

For structural equation 2:

$$\text{Kematian ibu} = \begin{cases} 0.16079 \text{ Prediksi (Wanita hamil yang beresiko tinggi)} \\ 0.31827 \text{ Pemberian tablet FE1} \\ -0.31754 \text{ Program K1} \\ 0.19763 \text{ Rumah tangga ber-PHBS} \\ 0.10434 \text{ Persalinan bantuan dukun} \end{cases}$$

Overall implementation path analysis model parameter estimation by GMM method in the case of maternal mortality in East Java suggests that the variable percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry the K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted shaman contribute directly or through variable pregnant women at high risk/complications against maternal mortality. Although the contribution made relatively small but can still be taken into consideration by the Government and relevant agencies in making the decision to create programs that aim to reduce the high maternal mortality.

## 4. Conclusion and remarks

- 1) Several conclusions can be drawn based on the discussions that have been described previously and referred to the purpose of the study, namely prediction/estimation parameters of path analysis can be completed by using the GMM. Because the path analysis consists of two structural equation model, the estimated completion can be done gradually.
- 2) Application of path analysis model parameter estimation by GMM method in the case of maternal mortality in East Java suggests that the variable percentage of pregnant women who received FE1 tablet, the percentage of pregnant women carry the K1 program, the percentage of households that behave clean and healthy living as well as a variable percentage of births assisted shaman contributes to maternal mortality either directly or via a variable percentage of pregnant women at high risk/complications.

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